UNIT-2

What is Data?

- Collection of data objects and their attributes

- An attribute is a property or characteristic of an object
  - Examples: eye color of a person, temperature, etc.
  - Attribute is also known as variable, field, characteristic, or feature
- A collection of attributes describe an object
  - Object is also known as record, point, case, sample, entity, or vector

Attributes

Objects

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Attribute Values

- Attribute values are numbers or symbols assigned to an attribute

- Distinction between attributes and attribute values
  - Same attribute can be mapped to different attribute values
    - Example: height can be measured in feet or meters

  - Different attributes can be mapped to the same set of values
    - Example: Attribute values for ID and age are integers
    - But properties of attribute values can be different
      - ID has no limit but age has a maximum and minimum value

Types of Attributes

- There are different types of attributes
  - Nominal
    - Examples: ID numbers, eye color, zip codes
  - Ordinal
    - Examples: rankings (e.g., taste of potato chips on a scale from 1-10), grades, height in {tall, medium, short}
  - Interval
    - Examples: calendar dates, temperatures in Celsius or Fahrenheit.
  - Ratio
    - Examples: temperature in Kelvin, length, time, counts

Properties of Attribute Values

- The type of an attribute depends on which of the following properties it possesses:
  - Distinctness: $= \; \neq$
  - Order: $< \; >$
  - Addition: $+ \; -$
  - Multiplication: $* \; /$

  - Nominal attribute: distinctness
  - Ordinal attribute: distinctness & order
  - Interval attribute: distinctness, order & addition
  - Ratio attribute: all 4 properties

Attribute Type	Description	Examples	Operations
Nominal	The values of a nominal attribute are just different names, i.e., nominal attributes provide only enough information to distinguish one object from another. (=, ≠)	zip codes, employee ID numbers, eye color, sex: {*male, female*}	mode, entropy, contingency correlation, χ^2 test
Ordinal	The values of an ordinal attribute provide enough information to order objects. (<, >)	hardness of minerals, {*good, better, best*}, grades, street numbers	median, percentiles, rank correlation, run tests, sign tests
Interval	For interval attributes, the differences between values are meaningful, i.e., a unit of measurement exists. (+, -)	calendar dates, temperature in Celsius or Fahrenheit	mean, standard deviation, Pearson's correlation, *t* and *F* tests
Ratio	For ratio variables, both differences and ratios are meaningful. (*, /)	temperature in Kelvin, monetary quantities, counts, age, mass, length, electrical current	geometric mean, harmonic mean, percent variation

Discrete and Continuous Attributes

- Discrete Attribute
  - Has only a finite or countably infinite set of values
  - Examples: zip codes, counts, or the set of words in a collection of documents
  - Often represented as integer variables.
  - Note: binary attributes are a special case of discrete attributes

- Continuous Attribute
  - Has real numbers as attribute values
  - Examples: temperature, height, or weight.
  - Practically, real values can only be measured and represented using a finite number of digits.
  - Continuous attributes are typically represented as floating-point variables.

Types of data sets

- **Record**
  - **Data Matrix**
  - **Document Data**
  - **Transaction Data**
- **Graph**
  - **World Wide Web**
  - **Molecular Structures**
- **Ordered**
  - **Spatial Data**
  - **Temporal Data**
  - **Sequential Data**
  - **Genetic Sequence Data**

Record Data

- Data that consists of a collection of records, each of which consists of a fixed set of attributes

Tid	**Refund**	**Marital Status**	**Taxable Income**	**Cheat**
1	Yes	Single	125K	**No**
2	No	Married	100K	**No**
3	No	Single	70K	**No**
4	Yes	Married	120K	**No**
5	No	Divorced	95K	**Yes**
6	No	Married	60K	**No**
7	Yes	Divorced	220K	**No**
8	No	Single	85K	**Yes**
9	No	Married	75K	**No**
10	No	Single	90K	**Yes**

Data Matrix

- If data objects have the same fixed set of numeric attributes, then the data objects can be thought of as points in a multi-dimensional space, where each dimension represents a distinct attribute

- Such data set can be represented by an m by n matrix, where there are m rows, one for each object, and n columns, one for each attribute

Projection of x Load	Projection of y load	Distance	Load	Thickness
10.23	5.27	15.22	2.7	1.2
12.65	6.25	16.22	2.2	1.1

Document Data

- Each document becomes a `term' vector,
  - each term is a component (attribute) of the vector,
  - the value of each component is the number of times the corresponding term occurs in the document.

	team	coach	play	ball	score	game	win	lost	timeout	season
Document 1	3	0	5	0	2	6	0	2	0	2
Document 2	0	7	0	2	1	0	0	3	0	0
Document 3	0	1	0	0	1	2	2	0	3	0

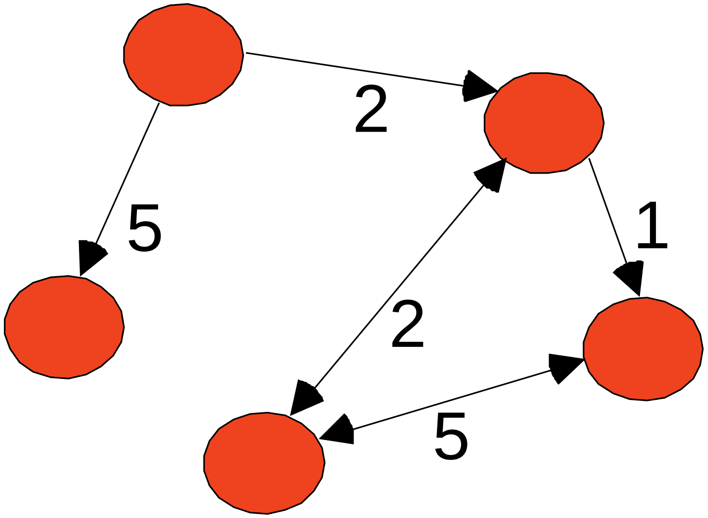
Transaction Data

- A special type of record data, where
  - each record (transaction) involves a set of items.
  - For example, consider a grocery store. The set of products purchased by a customer during one shopping trip constitute a transaction, while the individual products that were purchased are the items.

TID	*Items*
1	**Bread, Coke, Milk**
2	**Beer, Bread**
3	**Beer, Coke, Diaper, Milk**
4	**Beer, Bread, Diaper, Milk**
5	**Coke, Diaper, Milk**

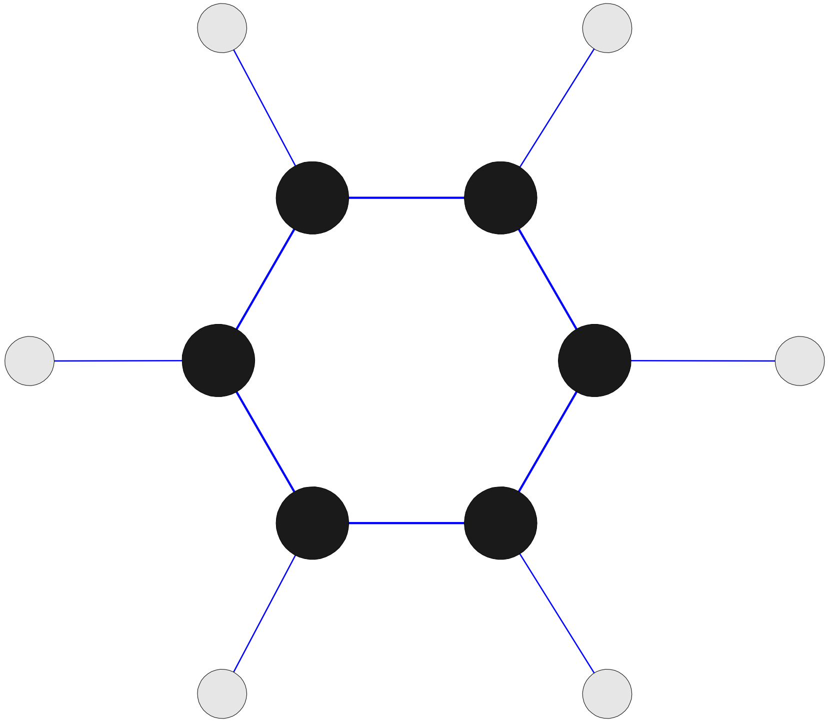
Graph Data

- Examples: Generic graph and HTML Links



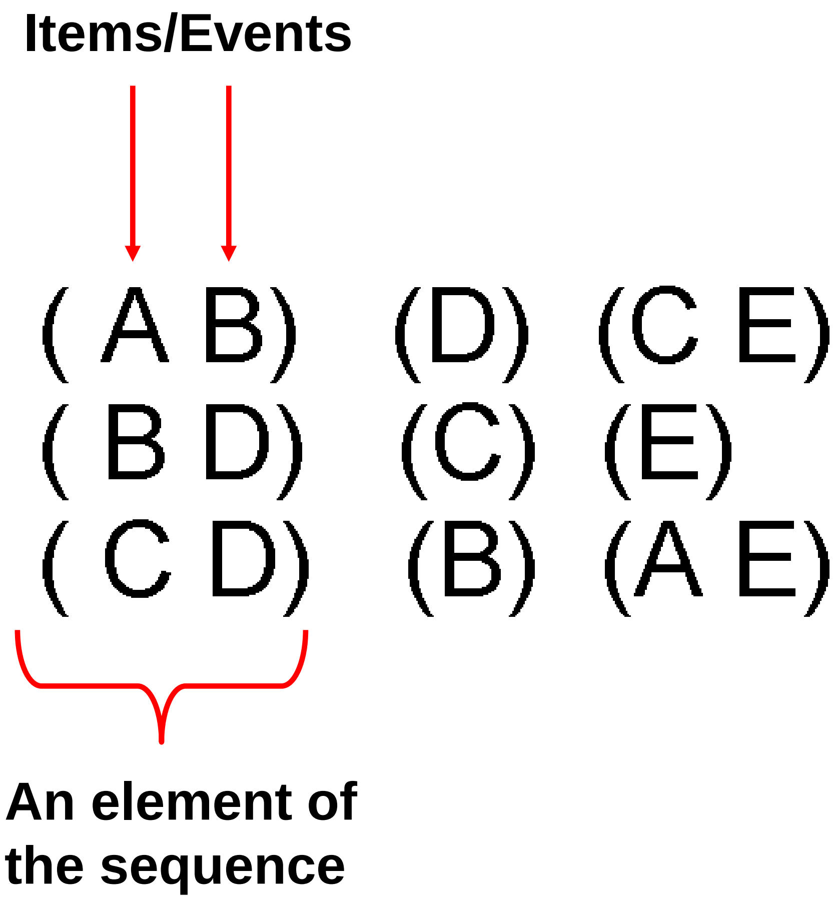
Chemical Data

- Benzene Molecule: C_6H_6



Ordered Data

- Sequences of transactions



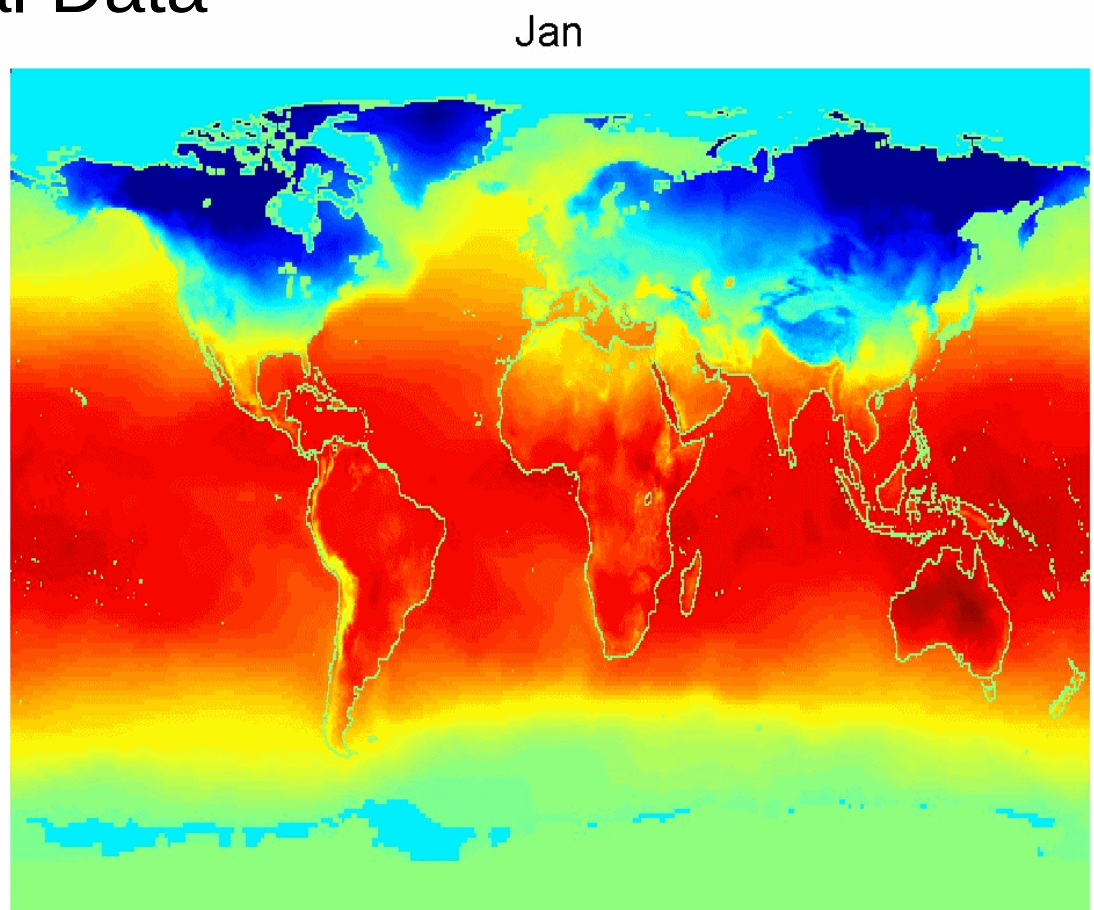
Ordered Data

- Genomic sequence data

```
GGTTCCGCCTTCAGCCCCGCGCC
CGCAGGGCCCGCCCCGCGCCGTC
GAGAAGGGCCCGCCTGGCGGGCG
GGGGGAGGCGGGGCCGCCCGAGC
CCAACCGAGTCCGACCAGGTGCC
CCCTCTGCTCGGCCTAGACCTGA
GCTCATTAGGCGGCAGCGGACAG
GCCAAGTAGAACACGCGAAGCGC
TGGCTGCCTGCTGCGACCAGGG
```

Ordered Data

- Spatio-Temporal Data



Average Monthly Temperature of land and ocean

Similarity and Dissimilarity

- Similarity
  – Numerical measure of how alike two data objects are.
  – Is higher when objects are more alike.
  – Often falls in the range [0,1]
- Dissimilarity
  – Numerical measure of how different are two data objects
  – Lower when objects are more alike
  – Minimum dissimilarity is often 0
  – Upper limit varies
- Proximity refers to a similarity or dissimilarity

Proximity b/w objects having a single attribute

Similarity/Dissimilarity for Simple Attributes

p and *q* are the attribute values for two data objects.

Attribute Type	Dissimilarity	Similarity
Nominal	$d = \begin{cases} 0 & \text{if } p = q \\ 1 & \text{if } p \neq q \end{cases}$	$s = \begin{cases} 1 & \text{if } p = q \\ 0 & \text{if } p \neq q \end{cases}$
Ordinal	$d = \frac{\lvert p-q \rvert}{n-1}$ (values mapped to integers 0 to $n-1$, where n is the number of values)	$s = 1 - \frac{\lvert p-q \rvert}{n-1}$
Interval or Ratio	$d = \lvert p - q \rvert$	$s = -d$, $s = \frac{1}{1+d}$ or $s = 1 - \frac{d - min\_d}{max\_d - min\_d}$

Table 5.1. Similarity and dissimilarity for simple attributes

Measures of proximity b/w objects that involve multiple attributes

- Dissimilarities b/w data objects:

  Distances: Euclidean Distance

  Manhattan Distance

  Minkowski Distance

- Similarities b/w data objects.

Euclidean Distance

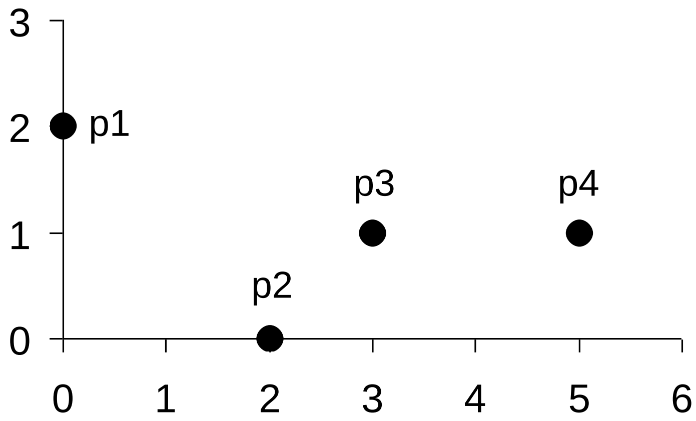
- Euclidean Distance

$$dist = \sqrt{\sum_{k=1}^{n} (p_k - q_k)^2}$$

Where n is the number of dimensions (attributes) and p_k and q_k are, respectively, the kth attributes (components) or data objects p and q.

- Standardization is necessary, if scales differ.

Euclidean Distance



point	x	y
p1	0	2
p2	2	0
p3	3	1
p4	5	1

	p1	p2	p3	p4
p1	0	2.828	3.162	5.099
p2	2.828	0	1.414	3.162
p3	3.162	1.414	0	2
p4	5.099	3.162	2	0

Distance Matrix

Minkowski Distance

- Minkowski Distance is a generalization of Euclidean Distance

$$dist = \left(\sum_{k=1}^{n} | p_k - q_k |^r \right)^{\frac{1}{r}}$$

Where r is a parameter, n is the number of dimensions (attributes) and p_k and q_k are, respectively, the kth attributes (components) or data objects p and q.

Minkowski Distance: Examples

- $r = 1$. City block (Manhattan, taxicab, L_1 norm) distance.
  - A common example of this is the Hamming distance, which is just the number of bits that are different between two binary vectors
- $r = 2$. Euclidean distance(L_2 norm)
- $r \to \infty$. "supremum" (L_{max} norm, L_∞ norm) distance.
  - This is the maximum difference between any component of the vectors
- Do not confuse *r* with *n*, i.e., all these distances are defined for all numbers of dimensions.

Minkowski Distance

point	x	y
p1	0	2
p2	2	0
p3	3	1
p4	5	1

L1	p1	p2	p3	p4
p1	0	4	4	6
p2	4	0	2	4
p3	4	2	0	2
p4	6	4	2	0

L2	p1	p2	p3	p4
p1	0	2.828	3.162	5.099
p2	2.828	0	1.414	3.162
p3	3.162	1.414	0	2
p4	5.099	3.162	2	0

L_∞	p1	p2	p3	p4
p1	0	2	3	5
p2	2	0	1	3
p3	3	1	0	2
p4	5	3	2	0

Distance Matrix

Common Properties of a Distance

- Distances, such as the Euclidean distance, have some well known properties.

  1. $d(p, q) \geq 0$ for all p and q and
     $d(p, q) = 0$ only if $p = q$. (Positive definiteness)
  2. $d(p, q) = d(q, p)$ for all p and q. (Symmetry)
  3. $d(p, r) \leq d(p, q) + d(q, r)$ for all points p, q, and r. (Triangle Inequality)

  where $d(p, q)$ is the distance (dissimilarity) between points (data objects), p and q.

- A distance that satisfies these properties is a metric

Similarity b/w Data Objects

- Similarities, also have some well known properties.

  1. $s(p, q) = 1$ (or maximum similarity) only if $p = q$.

  2. $s(p, q) = s(q, p)$ for all p and q. (Symmetry)

  where $s(p, q)$ is the similarity between points (data objects), p and q.

Examples of Proximity Measures

- Similarity Between Binary Vectors
  - Simple Matching Coefficient
  - Jaccard Coefficient
- Cosine Similarity
- Extended Jaccard Coefficient
- Correlation

Similarity Between Binary Vectors

- Common situation is that objects, *p* and *q*, have only binary attributes
- Compute similarities using the following quantities

M_{01} = the number of attributes where p was 0 and q was 1

M_{10} = the number of attributes where p was 1 and q was 0

M_{00} = the number of attributes where p was 0 and q was 0

M_{11} = the number of attributes where p was 1 and q was 1

- Simple Matching and Jaccard Coefficients

SMC = number of matches / number of attributes

$= (M_{11} + M_{00}) / (M_{01} + M_{10} + M_{11} + M_{00})$

J = number of 11 matches / number of not-both-zero attributes values

$= (M_{11}) / (M_{01} + M_{10} + M_{11})$

SMC versus Jaccard: Example

p = 1 0 0 0 0 0 0 0 0 0

q = 0 0 0 0 0 0 1 0 0 1

$M_{01} = 2$ (the number of attributes where p was 0 and q was 1)

$M_{10} = 1$ (the number of attributes where p was 1 and q was 0)

$M_{00} = 7$ (the number of attributes where p was 0 and q was 0)

$M_{11} = 0$ (the number of attributes where p was 1 and q was 1)

$$SMC = (M_{11} + M_{00})/(M_{01} + M_{10} + M_{11} + M_{00}) = (0+7) / (2+1+0+7) = 0.7$$

$$J = (M_{11}) / (M_{01} + M_{10} + M_{11}) = 0 / (2 + 1 + 0) = 0$$

Cosine Similarity

- If d_1 and d_2 are two document vectors, then

  $$\cos(d_1, d_2) = (d_1 \bullet d_2) / ||d_1||\ ||d_2||,$$

  where $\bullet$ indicates vector dot product and $|| d ||$ is the length of vector d.

- Example:

  d_1 = **3 2 0 5 0 0 0 2 0 0**

  d_2 = **1 0 0 0 0 0 0 1 0 2**

  $d_1 \bullet d_2$ = 3*1 + 2*0 + 0*0 + 5*0 + 0*0 + 0*0 + 0*0 + 2*1 + 0*0 + 0*2 = 5

  $||d_1|| = (3*3+2*2+0*0+5*5+0*0+0*0+0*0+2*2+0*0+0*0)^{0.5} = (42)^{0.5} = 6.481$

  $||d_2|| = (1*1+0*0+0*0+0*0+0*0+0*0+0*0+1*1+0*0+2*2)^{0.5} = (6)^{0.5} = 2.245$

  $\cos(d_1, d_2) = .3150$

Extended Jaccard Coefficient (Tanimoto)

- Variation of Jaccard for continuous or count attributes
  – Reduces to Jaccard for binary attributes

$$T(p,q) = \frac{p \bullet q}{\|p\|^2 + \|q\|^2 - p \bullet q}$$

Correlation

- Correlation measures the linear relationship between objects
- To compute correlation, we standardize data objects, p and q, and then take their dot product

$$\text{corr}(x,y) = \frac{\text{Covariance}(x,y)}{\text{Standard deviation}(x) * \text{Standard deviation}(y)} = \frac{S_{xy}}{S_x \cdot S_y}$$

where

$$\text{Covariance}(x,y) = S_{xy} = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

(whole population) $$\text{Standard deviation}(x) = S_x = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

$$\text{Standard deviation}(y) = S_y = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2}$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$$

Correlation is always in the range −1 to 1.

A correlation of 1 (−1) means that x & y have a perfect positive (negative) linear relationship.

Statistical Descriptions of Data

- Measuring the Central Tendency
- Measuring the Dispersion of Data

Measuring the Central Tendency

- Mean $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
  - Weighted arithmetic mean $\bar{x} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$
- Median: A holistic measure
  - Middle value if odd number of values, or
  - average of the middle two values if even
  - estimated by interpolation $median = L_1 + (\frac{n/2 - (\sum f)l}{f_{median}})c$
- Mode
  - Value that occurs most frequently in the data
  - Data sets with one, two, or three modes are respectively called Unimodal, bimodal, trimodal.
  - In general, a data set with two or more modes is multimodal.
  - Empirical formula: $mean - mode = 3 \times (mean - median)$
- Mid-Range is the average of the largest and smallest values in the set

Examples

$$30, 36, 47, 50, 52, 52, 56, 60, 63, 70, 70, 110.$$

Mean.

$$\bar{x} = \frac{30+36+47+50+52+52+56+60+63+70+70+110}{12}$$

$$= \frac{696}{12} = 58.$$

Median.

$$\frac{52+56}{2} = \frac{108}{2} = 54.$$

Mode:
Two Modes: 52,70

Mid-Range:
(30+110)/2=70

Variance and standard deviation.

$$\sigma^2 = \frac{1}{12}(30^2 + 36^2 + 47^2 \ldots + 110^2) - 58^2$$

$$\approx 379.17$$

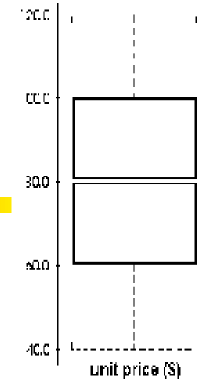
$$\sigma \approx \sqrt{379.17} \approx 19.47.$$

Measuring the Dispersion of Data

- Range of the set is the difference between the largest (max()) and smallest (min()) values.
- Quartiles, boxplots
  - Quartiles: Q_1 (25th percentile), Q_3 (75th percentile)
  - Inter-quartile range: IQR = $Q_3 - Q_1$
  - Five number summary: min, Q_1, M, Q_3, max
  - Five number summary approach is used in plotting a box plot
- Variance and standard deviation
  - Variance σ^2: (algebraic, scalable computation)
  - Standard deviation: σ

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2 = \left(\frac{1}{N}\sum_{i=1}^{N}x_i^2\right) - \bar{x}^2,$$

Boxplot Analysis



- Five-number summary of a distribution:

  Minimum, Q1, M, Q3, Maximum
- Boxplot
  - Data is represented with a box
  - The ends of the box are at the first and third quartiles, i.e., the height of the box is IRQ
  - The median is marked by a line within the box
  - Whiskers: two lines outside the box extend to Minimum and Maximum

Visualization

- Visualization is the conversion of data into a visual or tabular format so that the characteristics of the data and the relationships among data items or attributes can be analyzed or reported.

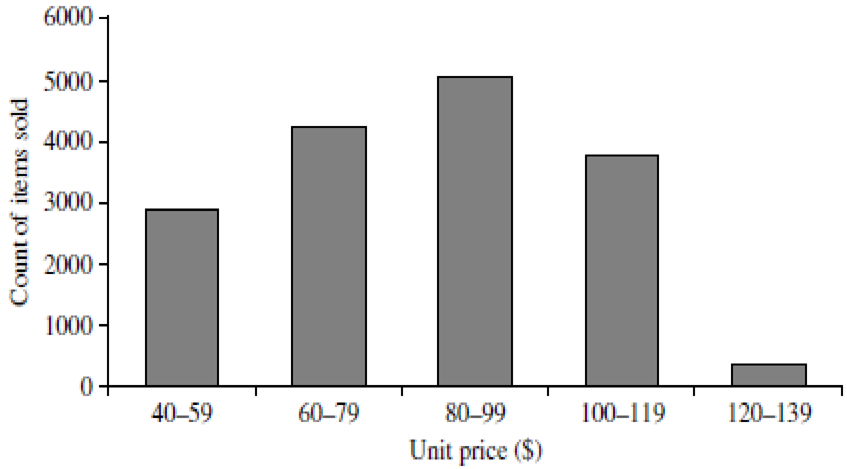
- **Visualization Techniques:**
  - Histograms
  - Box Plots
  - Scatter Plots
  - Piecharts

Histogram Analysis

- Graph displays of basic statistical class descriptions
  - Frequency histograms
    - A univariate graphical method
    - Consists of a set of rectangles that reflect the counts or frequencies of the classes present in the given data

A Set of Unit Price Data for Items Sold at a Branch of *AllElectronics*

Unit price ($)	*Count of items sold*
40	275
43	300
47	250
—	—
74	360
75	515
78	540
—	—
115	320
117	270
120	350



A histogram for the Table 2.1 data set.

Quantile Plot

- Displays all of the data (allowing the user to assess both the overall behavior and unusual occurrences)
- Plots quantile information
  - For a data x_i data sorted in increasing order, f_i indicates that approximately 100 f_i% of the data are below or equal to the value x_i

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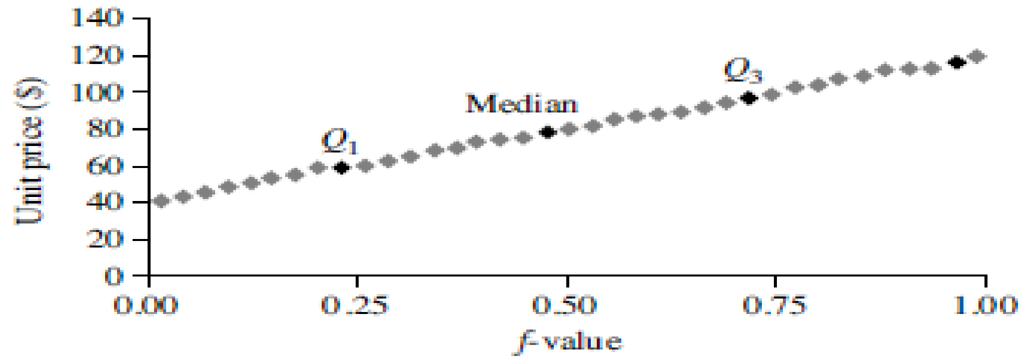

A quantile plot for the unit price data of Table 2.1.

Quantile-Quantile (Q-Q) Plot

- Graphs the quantiles of one univariate distribution against the corresponding quantiles of another
- Allows the user to view whether there is a shift in going from one distribution to another

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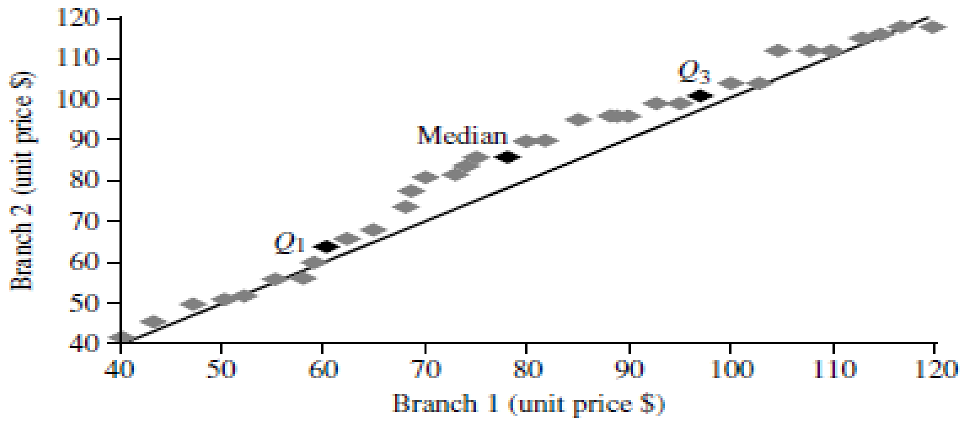

A q-q plot for unit price data from two *AllElectronics* branches.

Scatter plot

- Provides a first look at bivariate data to see clusters of points, outliers, etc
- Each pair of values is treated as a pair of coordinates and plotted as points in the plane

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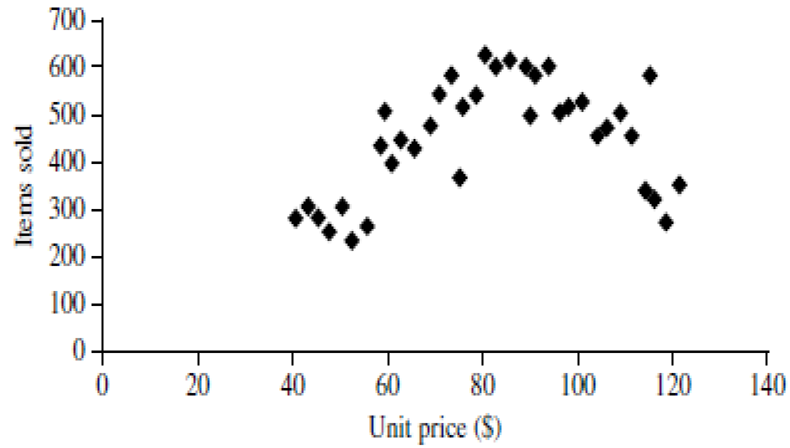

A scatter plot for the Table 2.1 data set.

Graphic Displays of Basic Statistical Descriptions

- Histogram: (shown before)
- Boxplot: (covered before)
- Quantile plot: each value x_i is paired with f_i indicating that approximately 100 f_i% of data are $\leq x_i$
- Quantile-quantile (q-q) plot: graphs the quantiles of one univariant distribution against the corresponding quantiles of another
- Scatter plot: each pair of values is a pair of coordinates and plotted as points in the plane